

Name: _____

50

- For full marks, show all of your work. "Guess and check" is not an appropriate method.
- Simplify your answers where applicable
- Unless otherwise specified, round answers to at least three significant digits.
- Mark values are indicated in square brackets to the left of each question.
- No electronic devices (including cell phones, smart watches, MP3 players, etc.) other than the Sharp EL-W535 calculator are permitted. Turn cell phones off.
- No items (calculators, rulers, erasers, etc.) may be shared between students.

[10] 1) A survey was done asking 600 people what their favourite colour is. The results are seen in the table below. If ONE person is selected at random, determine the probability that they:

		Colour			Totals
		Blue	Red	Green	
Age	Under 30	85	125	60	270
	Over 30	135	80	115	330
Totals		220	205	175	600

a) like red. (2 marks)

$$P(\text{Red}) = (205/600) = 0.3417$$

b) are over 30 and like blue. (2 marks)

$$P(\text{Over 30 \& Blue}) = (135/600) = 0.225$$

c) are under 30 or like green. (3 marks)

$$P(\text{Under 30 or Green}) = (270/600) + (175/600) - (60/600) = 0.6417$$

d) are over 30 given that they like blue. (3 marks)

$$P(\text{Over 30 | Blue}) = P(\text{Over 30 and Blue})/P(\text{Blue}) = (135/600)/(220/600) = 0.6136$$

[9] 2) Given a bushel of peppers containing 12 green peppers, 10 red peppers, and 8 yellow peppers. Determine the probability of:

a) pulling out a red or a green pepper. (3 marks)

$$P(\text{Red or Green}) = P(\text{Red}) + P(\text{Green}) = (12/30) + (10/30) = 11/15$$

b) not pulling out a green pepper. (3 marks)

$$P(\text{not Green}) = P(\text{Red or Yellow}) = [(10+8)/30] = 3/5$$

c) pulling out a yellow pepper, then pulling out another yellow pepper (without replacement). (3 marks)

$$P(\text{Yellow at first attempt}) = (8/30)$$

$$P(\text{Yellow at second attempt}) = (7/29)$$

$$P(\text{Yellow at first and Yellow at second}) = (8/30)*(7/29) = 0.0644$$

[9] 3) A company knows that 5% of people do not have a cell phone. If 6 people were chosen at random, determine the probability that:

a) exactly 2 of them do not have a cell phone. (2 marks)

$$P(X=2) = 6C2*(0.05)^2*(1-0.05)^4 = 0.0305$$

b) less than 2 of them do not have a cell phone. (4 marks)

$$P(X<2) = P(X=0) + P(X=1)$$

$$P(X=0) = 6C0*(0.05)^0*(1-0.05)^6 = 0.7351$$

$$P(X=1) = 6C1*(0.05)^1*(1-0.05)^5 = 0.2321$$

$$P(X<2) = 0.7351 + 0.2321 = 0.9672$$

c) between 2 and 4 of them do not have a cell phone, inclusive. (3 marks)

$$P(2 \leq X \leq 4) = P(X=2) + P(X=3) + P(X=4)$$

$$P(X=2) = {}^6C_2 * (0.05)^2 * (1-0.05)^4 = 0.0305$$

$$P(X=3) = {}^6C_3 * (0.05)^3 * (1-0.05)^3 = 0.0021$$

$$P(X=4) = {}^6C_4 * (0.05)^4 * (1-0.05)^2 = 0.0001$$

$$P(2 \leq X \leq 4) = 0.0305 + 0.0021 + 0.0001 = 0.0327$$

[3] 4) Given a Binomial Distribution with $p = 0.14$ and $n = 22$.

a) The graph of the distribution will be shifted towards the right.

T / F

b) The graph of the distribution will have 22 bars.

T / F

c) Normal Approximation can be used for this distribution.

T / **F**

[4] 5) Jamie has determined that 75% of his students skip class by the end of the term. Out of the 52 students Jamie has,

a) what is the expected number of students that will skip class by the end of term?

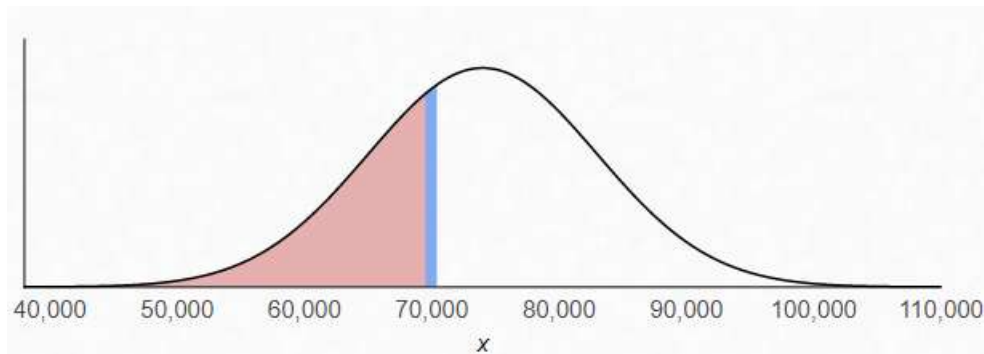
Expected number of students who would skip class = $0.75 * 52 = 39$ students

b) what is the standard deviation of students that will skip class by the end of term?

Standard deviation of students who will skip class = $(52 * 0.75 * (1 - 0.75))^{0.5} = 3.12$ students

[8] 6) Teachers earn a salary that is normally distributed with a mean of \$74,000 and a standard deviation of \$9,000.

a) What is the probability of a teacher earning less than \$70,000? Include a sketch. (4 marks)



Here $X = \$70,000$

$$Z = (70,000 - 74,000)/9,000 = -0.444$$

$$P(X < \$70,000) = P(Z < -0.444) = 0.3284$$

b) What is the probability of a teacher earning between \$70,000 and \$84,000? Include a sketch. (4 marks)



Here, $X_1 = \$70,000$

$$P(X < \$70,000) = 0.3284 \text{ (From above part)}$$

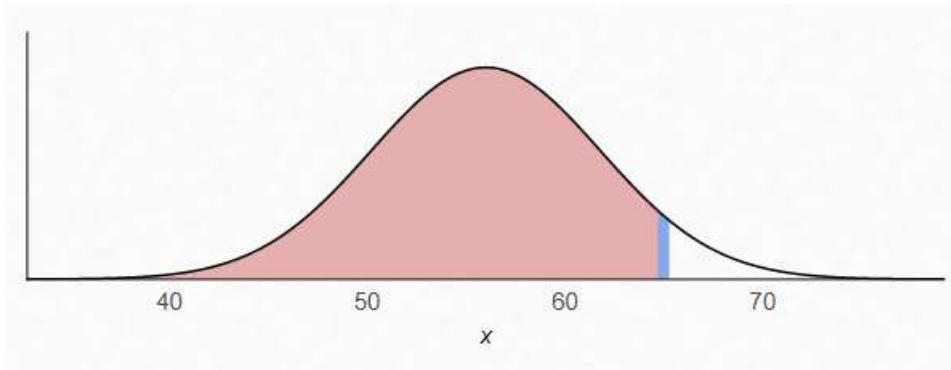
Here, $X_2 = \$84,000$

$$Z \text{ score} = (84,000 - 74,000)/9,000 = 1.11$$

$$P(X < \$84,000) = P(Z < 1.11) = 0.8667$$

$$\text{Hence, } P(\$70,000 < X < \$84,000) = 0.8667 - 0.3284 = 0.5383$$

- [7] 7) A study shows that 40% of people snore when they sleep. If 140 people are randomly selected, use *Normal Approximation* to determine the probability that exactly 65 of them snore when they sleep. Include a sketch.



The given distribution is binomial which would be approximated to normal distribution since the requisite conditions of np (i.e. $140 \cdot 0.4$) and $n(1-p)$ (i.e. $140 \cdot 0.6$) being greater than 5 are satisfied here.

$$\text{Mean people that snore} = 0.4 \cdot 140 = 56$$

$$\text{Standard deviation of average people that snore} = (56 \cdot 0.6)^{0.5} = 5.797$$

$$\text{Here } X = 65$$

$$\text{Z score} = (65 - 56) / 5.797 = 1.55$$

$$\text{Thus, probability that 65 would snore when they sleep} = P(Z < 1.55) = 0.9398$$

MTH 2223 Formulas

$$P(x) = \binom{n}{x} \cdot p^x \cdot (1 - p)^{n-x}$$

$$\mu = np \quad \sigma = \sqrt{np(1-p)}$$

$$z = \frac{x - \mu}{\sigma}$$